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Controlled disordered patterns and information transfer between coupled neural lattices with oscillatory states

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The problem of reproduction of spatial images by lattices of oscillating neural units is discussed. We consider that each neuron can be at rest or can oscillate with fixed frequency and that the neurons are coupled electrically by a resistor. Then one layer of neurons, one lattice, is coupled to another similar layer. It is shown that for strong enough interlattice interaction relative to the intralattice diffusion, the shape of the pattern on one lattice is determined uniquely by the image of the other. The reproduction of a stimulus shape is possible even when the number of interlattice couplings is much smaller than the number of neurons in either lattice. Moreover, the spatial features of the images do not depend on the features of the eigenexcitations of the neural lattices, which are discrete, active nonequilibrium media. [S1063-651X(98)11603-3]

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I. INTRODUCTION

The majority of neurons and neural groups (small and large neural systems) act as electric, or signal, or information generators. These may be subcritical and supercritical generators. All processing of the incoming signals occurs against the background of their own oscillatory activity. A number of experiments suggest that the information provided by the neurons and the neural groups is contained in the transformation of their mutual dynamics [1–4]. The representation and the reproduction of the information in neural assemblies is possible only due to the spatiotemporal modulation of the activity of the elements in the assembly. Dynamic representation of information in the neural systems may vary. For instance, it may be represented as the synchronization of neural groups with different phase lag [3,5,6], as transformation from spatiotemporal disorder to order in certain parts of the assemblies, and so on. For example, it is shown in [7] that sensory stimuli can lead to the changes in spatiotemporal patterns of activity rather than in average level of the activity.

In this paper we discuss an approach to information exchange between neural assemblies, namely, spatial controlling of patterns in neural lattices with oscillatory states.

We take as an example two coupled neural lattices in the simplest approximations: (i) an individual neuron may be in two modes (the state of rest and the regime of isochronous oscillations) [8,9] and (ii) the neurons are coupled resistively, diffusively [10,11]. We show that the spatial structure of the image from one lattice may be controlled by the pattern on the other. The initial conditions of the controlled lattice may include arbitrary spatial disorder. On the other hand it may happen that some coupling between the lattices fails to do so due to being broken or ill functioning. This is what we shall call a rarefied interlattice coupling.

Note that perception of stimulus occurs against the background of complex spatiotemporal dynamics of the neural lattice. In particular, the bistability of neurons (periodic oscillations or the state of rest) in the presence of intralattice coupling with the neighbors provides a stable regime of spatially irregular oscillations. The action of a stimulus does not suppress spatiotemporal chaos but modulates its spatial distribution so that the envelope patterns reproduce the shape of the stimulus. The patterns thus reproduced against the background of spatial disorder have nothing to do with the eigenmodes of neural lattice oscillations. With two lattices and two bistable states, the number of different stimuli that can be reproduced by the N^2 neuronal assembly at a suitable coupling between the elements is 2^{N^2}.

II. MODEL

Among the diversity of neuron models that are able to be in the state of rest and produce periodic oscillations we choose quite a simple one. We will model the neuron by a Van der Pol oscillator with subcritical bifurcation. Two different attractors coexist in the phase plane of such an oscillator. These are a stable fixed point and a stable limit cycle. For simplicity, we assume that the period of the oscillation does not depend on amplitude. In this case, one can describe the dynamics of such a neuron by the equation

$$\dot{u}_{jk} = -u_{jk}F(|u_{jk}|^2). \tag{1}$$

Here, u_{jk} is the complex amplitude of the oscillation in the j,k node or neuron of the lattice. $F(|u|^2) = 2a|u|^4 - a|u|^2$

$+1$, and $a>8$, allows coexistence of the state of rest and the periodic oscillation in the phase space of the system (1).

Two different formulations of the problem are possible. These are (i) mutual co-ordination of the patterns mapped by different lattices (in the case of reciprocal coupling between lattices) and (ii) reproduction of patterns through mutual, possibly rarefied interaction of the lattices. In this paper we investigate both cases. We consider two mutually coupled lattices described by the following equations:

$$\dot{u}_{jk}=-u_{jk}F(|u_{jk}|^2)+d(\Delta u)_{jk}+h_{jk}(v_{jk}-u_{jk}), \tag{2a}$$

$$\dot{v}_{jk}=-v_{jk}F(|v_{jk}|^2)+d(\Delta v)_{jk}+h_{jk}(u_{jk}-v_{jk}), \tag{2b}$$

$$j,k=1,2,\dots,N,$$

where d and h_{jk} account for intralattice and interlattice coupling, respectively. $(\Delta u)_{jk}=(\Delta u)_j+(\Delta u)_k-4u_{jk}$, $(\Delta u)_j=u_{j+1,k}-u_{j-1,k}$, and $(\Delta u)_k=u_{j,k+1}+u_{j,k-1}$. We restrict consideration to the case $h_{j,k}\geqslant 0$. We are going to consider the problem of reproduction of a *fixed* stimulus by a neural lattice as described by Eq. (2a), by taking v_{jk} as the external image, the stimulus image which is involved in the mutual interaction with disordered states u_{jk}. For some salient features in a case with only steady states available and a simple, cubic nonlinearity see Ref. [12].

III. IN-PHASE OSCILLATIONS

For a single lattice, Eq. (2), with $h_{jk}=0$, one can show (see [13] for details) that 2^{N^2} stable ''equilibria'' that set in-phase oscillations in the initial system may coexist in an autonomous lattice (system L_0) at sufficiently weak values of the intralattice, diffusive coupling. These oscillations exist for the values of the parameters in the region

$$D_{\mathrm{ch}}=\left\{d<\min\left[\frac{-f_{\min}}{4(r_0-r_{\min})},\frac{f_{\max}}{4(r_0+r_{\max})}\right],\quad a>8\right\},$$

where $r_{\min}$, $r_{\max}$, $f_{\min}$, $f_{\max}$ are, respectively, the abscissas and ordinates of the minima and maxima of the function $f(r)=-2ar^5+ar^3-r$ and r_0 is the largest root of the equation $f(r)=0$. The amplitude distribution of in-phase modes ''along'' the spatial coordinates is in one-to-one correspondence with $N\times N$ matrices consisting of a random set of two symbols. This means that the amplitude distribution of in-phase oscillations in the initial system may be extremely diverse: from simple (uniform, periodic, etc.) to irregular, chaotic in space.

A. Gradient features of the system

The system (2) with the interlattice coupling switched on may be rewritten as

$$\dot{u}_{j,k}=-\frac{\partial U}{\partial u^*_{j,k}},\quad \dot{v}_{j,k}=-\frac{\partial U}{\partial v^*_{j,k}}, \tag{3}$$

where the ''free energy'' functional U is

$$U=\sum_{j,k}\,[G(|u_{j,k}|^2)+G(|v_{j,k}|^2)+d(|u_{j+1,k}-u_{j,k}|^2$$
$$+|u_{j,k+1}-u_{j,k}|^2+|v_{j+1,k}-v_{j,k}|^2+|v_{j,k+1}-v_{j,k}|^2)$$
$$+h_{j,k}|u_{j,k}-v_{j,k}|^2],$$

$$G(|u|^2)=|u|^2-\frac{a}{2}|u|^4+\frac{2a}{3}|u|^6\geqslant 0.$$

Thus, Eq. (3) is a gradient system, and under arbitrary initial conditions its trajectories tend to one of the ''equilibria'' corresponding to one of the minima of the function U.

B. Manifold features of in-phase oscillations

The existence of a manifold of in-phase oscillations equilibria follows immediately from the system (2):

$$S=\{\varphi_{j,k}=\psi_{j,k}=\theta_0=\text{const},\quad j,k=1,2,\dots,N\},$$

where the phases $\varphi_{j,k}$ and $\psi_{j,k}$ are related to the variables of the system (2):

$$u_{j,k}=r_{j,k}e^{i\varphi_{j,k}},\quad v_{j,k}=\rho_{j,k}e^{i\psi_{j,k}}$$

By linearizing the system (2) in the vicinity of S and using the Gershgorin theorem [14] for the matrix of the linearized system it can be shown that the manifold S is *locally* stable.

C. Mutual synchronization of oscillations

Let us now show that due to the interlattice coupling the oscillations in either lattice are mutually synchronized on S if the inequality

$$h_{j,k}\geqslant\frac{7a-20}{40} \tag{4}$$

is fulfilled, i.e., when the interlattice couplings exceed some critical values. From Eq. (2) we find that, on the manifold S, the equations describing the dynamics of amplitudes have the form

$$\dot{r}_{j,k}=-r_{j,k}F(r_{j,k})+d(\Delta r)_{j,k}-h_{j,k}(r_{j,k}-\rho_{j,k}),$$
$$\dot{\rho}_{j,k}=-\rho_{j,k}F(\rho_{j,k})+d(\Delta\rho)_{j,k}+h_{j,k}(r_{j,k}-\rho_{j,k}). \tag{5}$$

Let us introduce new variables:

$$x_{j,k}=u_{j,k}-v_{j,k},\quad y_{j,k}=u_{j,k}+v_{j,k}.$$

Then Eq. (5) yields

$$\dot{x}_{j,k}=d(\Delta x)_j-\sigma x_{j,k}+d(\Delta x)_k-\frac{a}{8}x_{j,k}Q_{j,k}, \tag{6}$$

where

$$\sigma\equiv 4d+2h_{j,k}-\frac{7a-20}{20},$$

$$Q_{j,k}\equiv x^4_{j,k}+10x^2_{j,k}y^2_{j,k}-2x^2_{j,k}-6y^2_{j,k}+5y^4_{j,k}+\frac{14}{5}.$$

From Eq. (6) it follows that there exists in the phase space of the system (6) a manifold of in-phase oscillations

$$M=\{x_{jk}=0;\quad j,k=1,2,\ldots,N\},$$

which is *globally* stable. To prove this we introduce the Lyapunov function

$$V=\sum_{j,k}\frac{x_{j,k}^2}{2}.$$

Due to Eq. (4), the derivative of the function V has the form

$$\dot{V}=-\sum_{j,k}\left(P_{j,k}+\frac{a}{8}x_{j,k}^2Q_{j,k}\right),$$

where

$$P_{j,k}\equiv-dx_{j,k}(\Delta x)_j\sigma x_{j,k}^2-d(\Delta x)_kx_{j,k},$$

hence all functions $Q_{j,k}$ are positive definite. The function $P=\Sigma_{j,k}P_{j,k}$ is also positive definite. Indeed, introducing the vector

$$\mathbf{Z}=(z_1,z_2,\ldots,z_{N^2}),$$

where $z_1=x_{11}$, $z_2=x_{12}$, $\ldots$, $z_{N^2}=x_{NN}$, P may be written in the following form:

$$P=\sum_{i,j=1}^{N^2}a_{ij}z_iz_j=\mathbf{Z}^TA\mathbf{Z},$$

where $a_{ij}=a_{ji}$, T denotes transposition, and $A=\|a_{ij}\|$ is a square symmetric $N^2\times N^2$ matrix. The square form of P is positive definite if its eigenvalues are positive (see, for example, [15]). Using Gershgorin's theorem [14] it can be seen that the union of the Gershgorin disks in the complex plane is located in the right-side half plane, if the inequality (4) is fulfilled. Thus, the derivative $\dot{V}$ is a negative definite function and, consequently, the manifold M is *globally asymptotically* stable for all in-phase motions. This means that the different amplitude distributions of in-phase motions in two coupled lattices evolve to the amplitude patterns of identical spatial structure. This phenomenon may be interpreted as the *mutual* synchronization of oscillations in the two coupled lattices.

IV. REPLICATION OF A STIMULUS IN MUTUALLY COUPLED LATTICES

Let us see how the mutual synchronization of oscillations may lead to the replication by the disordered lattice of a given stimulus carried by the other patterned lattice.

A. Replication phenomenon

Let the initial conditions be the following: (i) The first lattice produces the in-phase motions with spatially disordered, but steady distribution of the oscillation amplitude (see, for example, Fig. 1). (ii) The amplitude distribution of in-phase motions in the second lattice is a given pattern (stimulus), which also corresponds to equilibrium of the independent lattice. We take as the stimulus the letter "*N*," with its black core being composed of oscillators that have been excited (having a finite amplitude), while the background outside the letter *N* contains the oscillators practically at rest (having vanishing amplitude). Let the interlattice interaction be switched on and the values of coupling coefficients satisfy the inequality (4). After a transient process the synchronization of oscillations occurs because the manifold *M* is *globally* stable, but what spatial structure of the terminal amplitude distribution will appear in the lattices? Figures 1–3 illustrate the result of the interaction by using $d=0.06$ and $h=0.6$ for the three cases while the shape of nonlinearity F (the value of the parameter a) describing a local oscillator is different. In the figures, the left pair of pictures corresponds to initial amplitude distributions, and the right pair to the terminal patterns obtained by numerical integration of the system (5). As already stated, the white color in the figures denotes the oscillators close to the state of rest, while the excited oscillators with maximum amplitude are shown in black. Then in all three figures the lattice initially disordered replicates the shape of the stimulus contained in the other lattice, although the terminal patterns of Figs. 1 and 2 preserve features of disorder either in the core of the letter (Fig. 1) or in the background (Fig. 2). The terminal patterns of Fig. 3 show the possibility of a rather faithful replication of the stimulus. What scenario and which system parameters are responsible for the *quality* of replication? Let us look, first, at Fig. 4 where the interaction occurs in the presence of another letter, "*V*," in the disordered lattice (which, for example, has been earlier replicated on this lattice), with the parameters taken the same as for the faithful replication shown in Fig. 3. We find that the terminal patterns in this case have a significant trace of the letter "*V*" in the core of the stimulus. Thus, these four examples show that replication

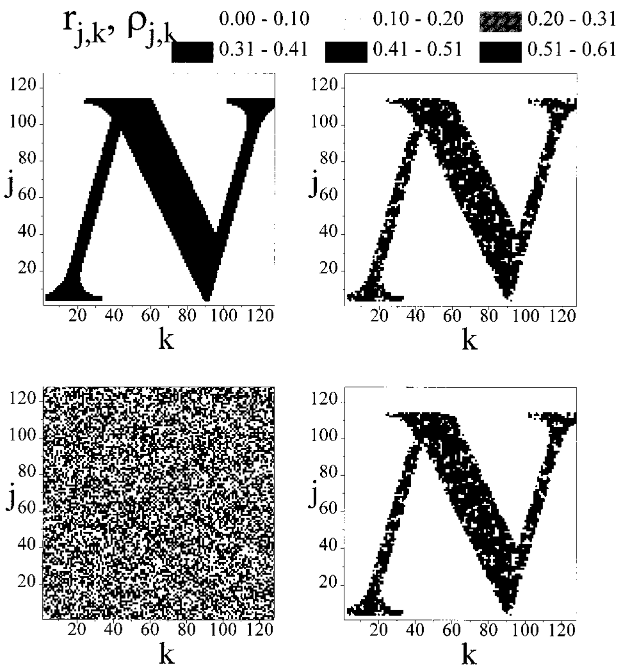

FIG. 1. Replication of the patterned stimulus with distortions in the core of the replicated image. Initial amplitude distributions (left pair), terminal patterns (right pair). Parameter values: $a=10$, $d=0.06$, $h_{jk}=h=0.6$.

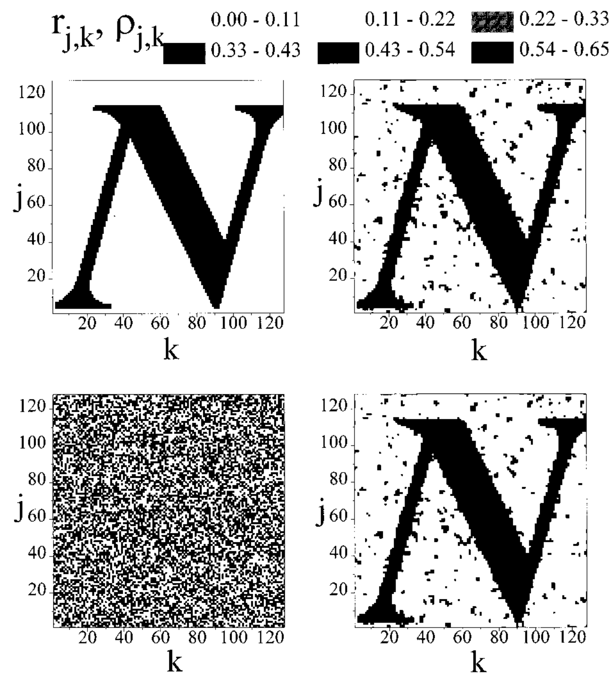

FIG. 2. Replication of the patterned stimulus with distortions in the background of the replicated image. Parameter values: $a=11$, $d=0.06$, $h_{jk}=h=0.6$.

is sensitive both to the system parameters and to the "properties" of initial distributions. The explanation of why an initially disordered lattice is able to reproduce the pattern of another lattice and to control the quality of this reproduction is given in the following section.

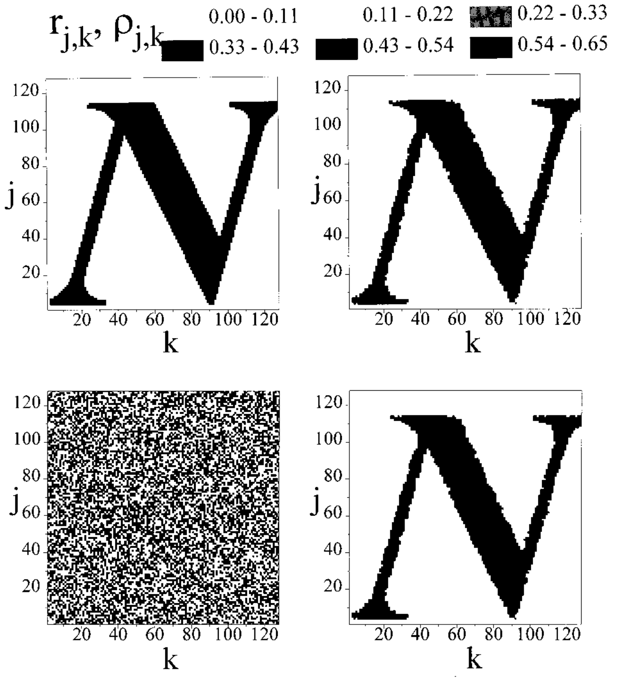

FIG. 3. Faithful replication of the patterned stimulus. Parameter values: $a=10.4$, $d=0.06$, $h_{jk}=h=0.6$.

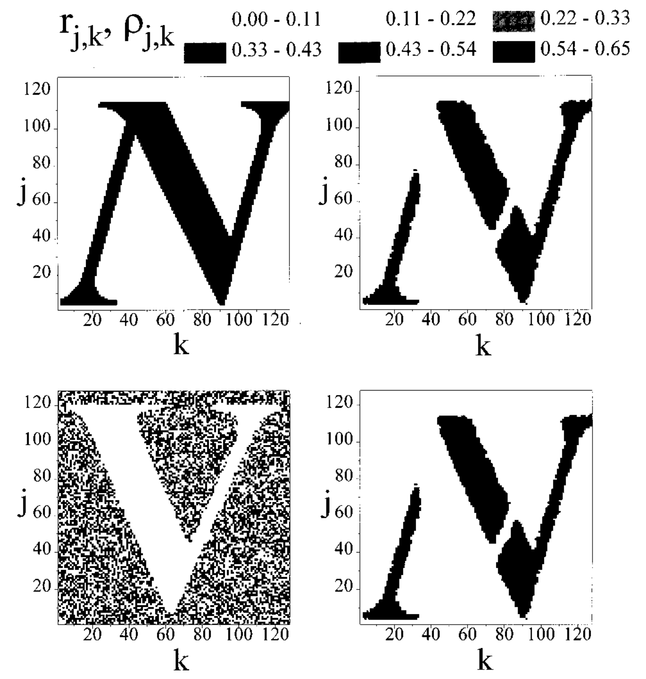

FIG. 4. Replication of the patterned stimulus when the initial disorder contains another pattern (letter "V"). Parameter values: $a=10.4$, $d=0.06$, $h_{jk}=h=0.6$.

B. Dynamical origin of replication

Note, first of all, that the intralattice, diffusive coupling d between the oscillators in each lattice must be small enough (in region D_{ch}). The 2^{N^2} possible equilibrium amplitude distributions of in-phase motions allows us to exhibit independent lattice patterns of widely arbitrary spatial structure. Since a local neuron is bistable, the possible amplitude of its oscillations for small d can be localized in the so-called absorbing domains (see [13]), i.e., small regions close to the rest state, O^0, and to the excited state, O^+, of each independent oscillator.

Let us consider the stimulus as a number of clusters (we define a cluster as a group of rather large number of neighboring oscillators having the same amplitude — close either to the rest state or to the excited state). For instance, the stimulus in the Figs. 1–4 (letter "*N*") has two clusters (core and background). A completely disordered pattern does not have clusters. Then the elements of a cluster are located in the absorbing domains close to the states O^0, if this is a cluster of oscillators at rest, or close to O^+, if this is a cluster of excited oscillators (for instance, the elements of the homogeneous pattern having one cluster are located in either O^0 or O^+, precisely), while the elements of a disordered pattern are located rather far from O^0 or O^+ but within the absorbing domains.

The second point to be noted is that the interlattice couplings $h_{j,k}$ should be strong enough to obtain the synchronization of the amplitude patterns, i.e., condition (4) must be fulfilled. Thus, any neuron (j,k) of one lattice has a weak interaction with its neighbors, $d \neq 0$, while it has a strong interaction with the corresponding neuron of the other lattice, $h_{j,k} \neq 0$, hence $\inf_{j,k} h_{j,k} = h_m \gg d$ is satisfied. This allows us to describe the dynamics of the amplitude of any pair of

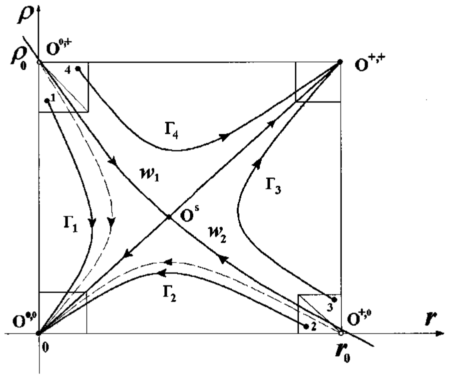

FIG. 5. Phase portrait of the auxiliary system (7). The four rectangles denote the domains where the initial conditions for the system should be situated. The trajectories $\Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4$ show the possible routes of replication as the result of *competition* of a pair of oscillators taken from different lattices.

oscillators (one taken from the first lattice r and the other ρ from the second), coupled by $h_{j,k}$, separately. The equations are

$$\dot{r} = -rF(r) - h_{j,k}(r-\rho),$$

$$\dot{\rho} = -\rho F(\rho) + h_{j,k}(r-\rho). \tag{7}$$

Figure 5 illustrates the qualitative phase portrait of the system (7). There are two stable equilibrium nodes $O^{0,0}(0,0)$ and $O^{+,+}(r_0, r_0)$ corresponding to the rest and excited states of the oscillators, and the saddle point O^s whose incoming separatrices are denoted by w_1 and w_2. The initial conditions for this system should be associated with equilibrium patterns of independent lattices. Since we use system (7) for any pair of oscillators all possible initial points are located in the absorbing domains, near one of the points $O^{0,0}$, $O^{+,+}$, $O^{0,+}$, $O^{+,0}$ (see Fig. 5). Taking the initial conditions within the absorbing domains we see the influence of the small intralattice diffusive coupling d in such a way that the state of all other elements of the lattice only determines the position of the initial point within the domain. Then, if both oscillators from a pair have been near the rest state or have been excited before the interaction (initial conditions should be taken in the domains near points $O^{0,0}$ or $O^{+,+}$, respectively) they do not change their states (do not leave the corresponding domains) after the interlattice interaction is switched on. Then, if the oscillators have been in different states (the initial point lies within the domains near $O^{0,+}$ or $O^{+,0}$ [these points are not equilibrium states for the system (7)], Fig. 5) they change their states and become synchronized as result of interlattice coupling. Possible routes of this process are shown in Fig. 5 by the trajectories $\Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4$. They differ by the location of the initial point relative to the separatrices w_1, w_2. Thus, pattern interaction is reduced to the *competition* of oscillations in the corresponding pairs of oscillators in the coupled lattices.

The replication of a stimulus can be also explained as follows. Let the first lattice described in Eq. (7) by r contain a patterned stimulus while the second lattice ρ is disordered. Consider one oscillator from a rest cluster of the stimulus and corresponding excited oscillators of the disordered lattice (point "1" in Fig. 5). The excited elements of this pair tend to the rest state by the route Γ_1. Analogously, the rest elements of the disorder in interaction with the excited cluster tend to the excited state (trajectory Γ_3). Thus, as a result of intralattice interaction the lattice of disordered oscillators *copies* the spatial structure of the stimulus with faithful replication.

As we mentioned above an oscillator from the disordered pattern stays rather far from the steady points of an independent oscillator while an oscillator from a cluster is very close to these states. Hence, the initial conditions are located near two *angles* of absorbing domains for a pair of oscillators when one of them is taken within a cluster and another from the disordered state (points "1" and "3" in Fig. 5). These angles are associated with the edge of the absorbing domain for one oscillator and the origin of the domain for another. This is the major ingredient in the evolution process, essential for replication. (It fails only for oscillators located near the boundaries of the clusters, hence the distortions of the replicated images even when the replication is rather faithful.) The second ingredient concerns the mutual arrangement of the separatrices w_1 and w_2 relative to the absorbing domains. It depends on the shape of the nonlinearity and on the strength of the interlattice coupling $h_{j,k}$. Faithful replication for all kinds of stimuli occurs when the separatrices lie as close as possible to the bisectors (Fig. 5) of the absorbing domain rectangles (or separate the angles where the initial points are located). If the separatrices intersect the domains near one of the angles or do not intersect them at all we will have traces of disorder in the rest cluster (Fig. 1) or in the excited cluster (Fig. 2) of the replicated image.

Notice also that if the initially disordered lattice contains a cluster (letter "*V*" in Fig. 4) the condition for faithful replication is not fulfilled (point "3" stays below the separatrix w_1) for some pairs of oscillators. Then these pairs do not copy the features of the stimulus but retain the features of the disorder (letter "*V*"). Thus replication in this case is incomplete.

Needless to say, there is no faithful replication process for vanishing intralattice diffusion $d=0$. In this case the initial conditions are located directly in the points $O^{+,0}$, $O^{0,+}$ (Fig. 5) and no matter what spatial structure of interacting patterns and interlattice coupling exist (no absorbing domains in this case) they are always mapped to the rest state (as shown in Fig. 5 by the dashed trajectories) or to the excited state (if the separatrices w_1, w_2 lie below the points $O^{+,0}$, $O^{0,+}$ in Fig. 5) for each pair of oscillators. This excludes the possibility of faithful replication.

In summary the main features of the replication process described above are as follows:

(i) The replication of a patterned stimulus by a disordered lattice occurs through mutual interaction of strongly coupled lattices with a weak but not vanishing diffusion in each lattice ($0 < d \ll h_m$).

(ii) The interlattice interaction can be described as a kind of *competition* of the amplitude states of each pair of strongly coupled oscillators taken independently in a point (j,k) of both lattices. The initial condition for this competition is defined uniquely by the spatial structure of the image and the disorder for nonvanishing intralattice coupling.

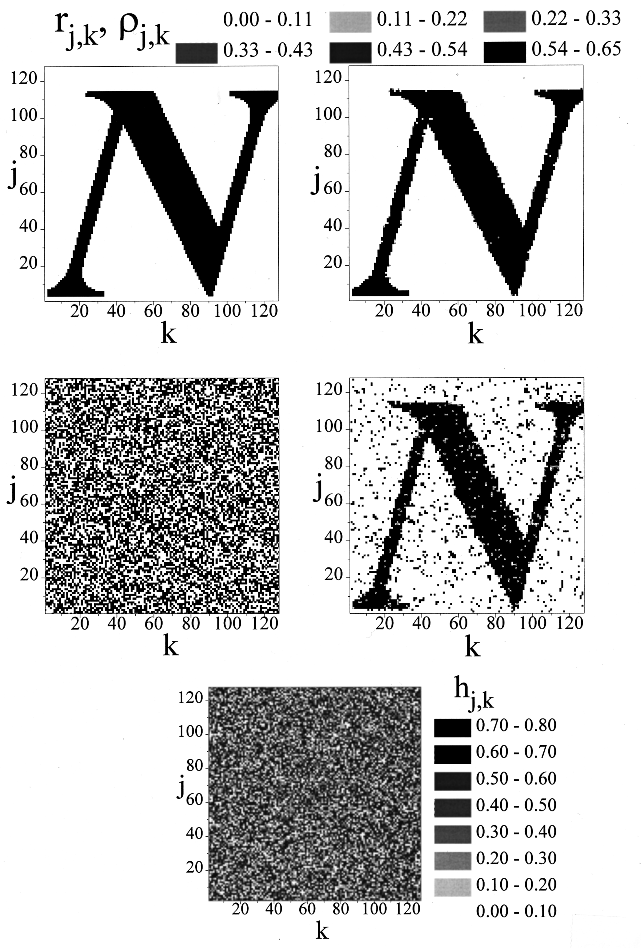

FIG. 6. Replication of the patterned stimulus in the case of rarefied coupling between the lattices. The values of $h_{j,k}$ are randomly distributed (bottom picture and corresponding grey scale indicating $0<h_{jk}<0.8$). Parameter values: $a=10.4$, $d=0.06$.

(iii) The second lattice or ''raw material'' ready for replication should be quite disordered or at least it should not contain clusters of oscillators. This provides an advantage to a patterned stimulus relative to a disordered distribution in the competition hence gives the possibility of faithful replication.

C. Replication for rarefied coupling between the lattices

Previously we have considered the problem of mutual interaction of two lattices when all interlattice coupling coefficients $h_{j,k}$ are equal to each other and taken above some threshold. (The threshold obtained in Sec. III gives only the upper estimated value of real threshold, which is varied for different patterns and depends on the lattice parameters and spatial structures of the initial distributions.) This is the case of *homogeneous* coupling between the lattice layers. Now let us see what happens if there is a number of interlattice couplings $h_{j,k}$ below threshold or simply broken. (i) Would the replication of a given stimulus still be possible for such rarefied couplings? (ii) What about the ''quality'' of replication?

Let us turn, first, to Fig. 6, which illustrates the results of numerical integration of the equations (7) for a case of rarefied coupling while all other parameters are taken the same as for the faithful replication shown in Fig. 3. The coefficients of the matrix $h_{j,k}$ are randomly distributed within the interval $[0,0.6]$. Thus, we have some couplings above threshold, others below threshold, and a number of couplings completely broken, hence missing. One can see that the replication actually still occurs here but, as expected, the replicated image is rather imperfect relative to the case of homogeneous coupling (Fig. 3).

Following the arguments given in Sec. IV B we can say the following. The elements in the points (j,k) for which the coefficients $h_{j,k}$ exceed the threshold interact in the same manner as described for the case of homogeneous coupling; i.e., the disordered oscillators tend to the state defined by the elements of the stimulus. The pairs of oscillators with broken or too weak $h_{j,k}$ cannot be described by the system (7) because the condition $h_{j,k} \gg d$ is not fulfilled and they cannot be considered independent of the other lattice elements. If before the interaction the oscillators of such a pair have been in different states (near state of rest or near the excited state) they cannot change their states because the coupling d with the neighbors is too weak (the magnitude of d has been chosen, namely, to provide the possibility of existence of all $2^{(N-1)^2}$ steady states of the lattice for fixed state of a given oscillator). Thus, in the points (j,k) of broken coupling $(h_{j,k} \to 0)$ the replicated image has distortions that we can see in the terminal patterns of Fig. 6. The oscillators of the disordered lattice for which $h_{j,k}$ is not vanishing but does not exceed the threshold can both synchronize with the stimulus by means of the cooperative action of small d and not too large $h_{j,k}$. Either they replicate features of the stimulus or they tend to a state located somewhere between the state of rest and the excited state but outside the absorbing domains of these states. Notice that the terminal patterns are equilibria for the whole system (the two coupled lattices as a composite system) because the system is gradient. However, they are not equilibrium states for each lattice taken separately in contrast to the case of homogeneous coupling.

In summary, the rarefied coupling also provides the ability to replicate a given patterned stimulus in the disordered lattice through the mutual interaction of the two lattices. Since the system cannot ''overcome'' the broken coupling by means of neighbors action (the coupling d in each lattice is very small) the replicated image will always have distortions where the couplings are broken.

D. Replication of a stimulus on lattices of oscillators with phases arbitrary distributed

We have considered the mutual interaction of two coupled lattices assuming that all oscillators in either lattice are in-phase. It has also been shown in Sec. III that the in-phase motions are *locally* stable, i.e., all small disturbances imposed on the oscillators phases will asymptotically decrease to the phase synchronization mode in the process of evolution. Here we illustrate the replication phenomenon, which occurs through the synchronization of oscillator *amplitudes,* when the phases of oscillators have been initially randomly distributed along the lattices.

Let us take the initial amplitude patterns and the system parameters corresponding to the faithful replication for in-phase motions (Fig. 3) and randomly distributed phases of all oscillators within the interval $(-1,1)$. Integrating Eq. (2) we

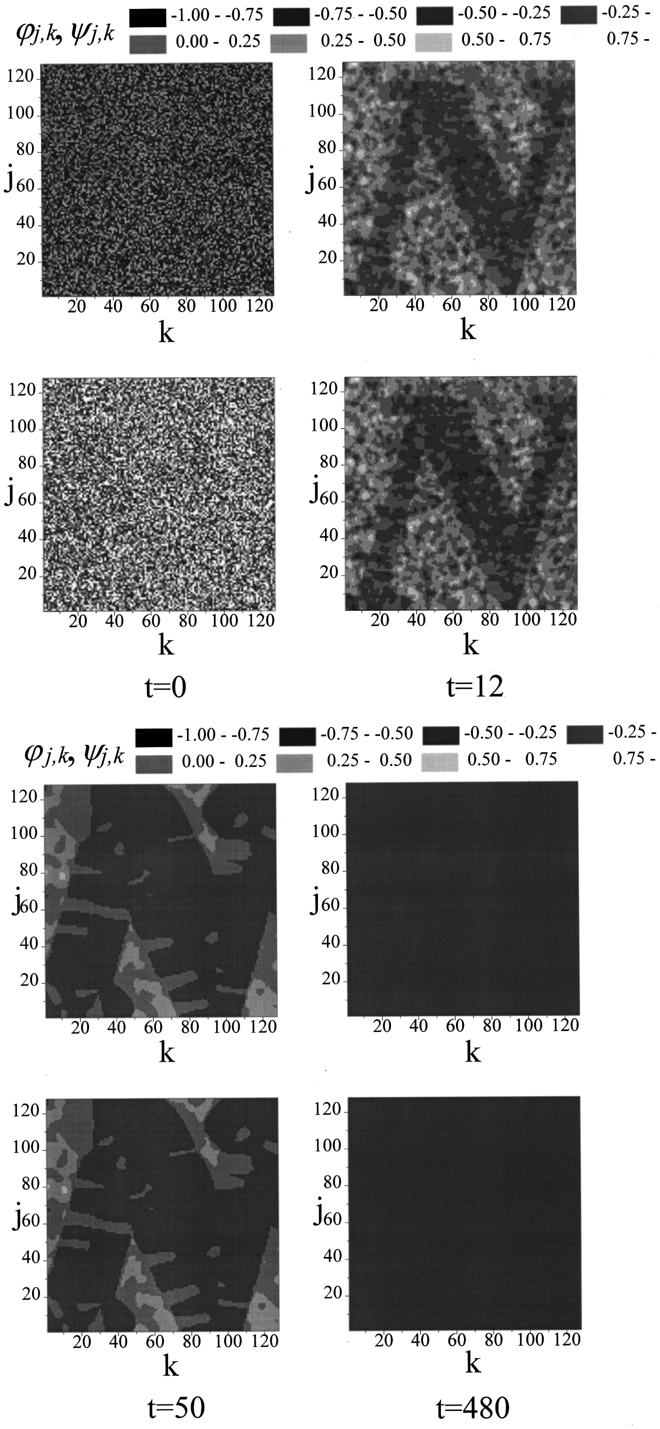

FIG. 7. Replication of the patterned stimulus on the mutually coupled lattices of oscillators with arbitrary distributed phases. Evolution of phase distributions. Vertical pairs of pictures show intermediate stages in the evolution. Terminal patterns correspond to the phase synchronization mode. Parameter values: $a=10.4$, $d=0.06$, $h=0.6$.

find that amplitude patterns synchronize rather quickly ($t\approx 12$) (replicated amplitude patterns are shown in Fig. 3) where the initially disordered lattice reproduces the stimulus given by the second lattice. The evolution of the phases of oscillators has a longer time scale, and Fig. 7 illustrates the steps of this evolution. The terminal state of all oscillators is close to the phase synchronization mode (the last pair of pictures in Fig. 7). The previous steps shown in Fig. 7 illustrate how the system reaches this state. This process involves forming *phase clusters* (we define a phase cluster as a group of neighbor oscillators with equal phase) which grow following intralattice interaction (small clusters are absorbed by big ones) and, finally, we have all oscillators in-phase. (Further details on phase cluster interaction are given in Refs. [13,16].) Notice that the phase patterns on the intermediate stages clearly copy the main features [the contours of the letter ''*N*'' (Fig. 7)] of the amplitude patterns already synchronized (Fig. 3). Thus, the coupled lattices of oscillators whose phases have been arbitrarily distributed are able to replicate a patterned stimulus carried by the amplitudes while the phases of all oscillators tend to the phase synchronization mode.

V. CONCLUSION

We have investigated how control and synchronization phenomena occur in a system of two coupled neural networks, i.e., two lattice layers composed of active, bistable oscillating neurons. The model discussed here is a particular example using control and synchronization of spatiotemporal images to transfer and process analog information between different neural assemblies or different parts of the cortex. We used for this model quite a simple description of the bistable behavior of the neurons [9]. We also assumed that the coupling between neurons inside each lattice is weak relative to the interlattice coupling. Such an assumption is common when modeling the cooperative dynamics of neural assemblies (see, for example, [10]).

The most important results of our study are the following: (i) for strong enough coupling between lattices the spatial pattern on one neural lattice controls completely the image pattern replicated on another one; (ii) such control is realized even when the number of connections between assemblies is much smaller than the number of neurons in each lattice; (iii) the observed phenomena also occur when the spatial distribution of interlattice connection is disordered in space. The robustness of the observed control and synchronization phenomena permits us to safely say that similar results are expected for more realistic models, which may take into account (1) the spiking-bursting behavior of neurons described by a conductance-based model (Hodgkin-Huxley–like model) and/or (2) the nonlocal coupling between neurons in the lattices, two items for future work.

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